GFZ POTSDAM

Fuzzy logic as an alternative tool for the interpretation of sea-level indicators with respect to glacial-isostatic adjustment

Volker Klemann, Detlef Wolf

Geodesy and Remote Sensing, GeoForschungsZentrum Potsdam, Germany (volkerk@gfz-potsdam.de)

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Summary

We present a novel procedure based on fuzzy logic to infer the process of glacial-isostatic adjustment from **sea-level indicators (SLIs)**. The depositional conditions of SLIs give evidence of the **relative sea-level (RSL)** height at the time of deposition. This allows us to validate models predicting glacially induced sea-level variations. In comparison to former studies, the new method has the advantage that it allows a more objective interpretation of the SLIs. Moreover it permits us to study regions where SLI data are of minor quality.

As an example of the method, we reassess the decay time of land uplift in the Richmond-Gulf region. The decay time is a key quantity to determine the mantle viscosity below central Canada: ***T* = 5.8 ka**.

Conventional procedure

Comparison of observational RSL height, derived from SLI, with predicted RSL height

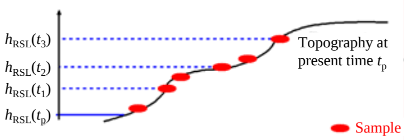
Novel procedure

Comparison of observational SLI height with predicted SLI-height range defined by depositional conditions

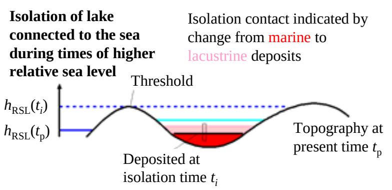
Reference

Klemann, Wolf , 2006. Using Fuzzy logic for the analysis of sea-level indicators with respect to glacial-isostatic adjustment, Pure Appl. Geophys., **accepted**.

SLIs, such as fossil samples or sediments, give evidence of the retreat of RSL height, h_{RSL}, during the last deglaciation (10,000 a until present time).

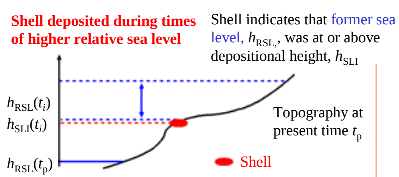


In Fennoscandia, available SLIs, mainly **isolation basins** (right), are of excellent quality, and allow an accurate determination of former sea level. Therefore, the inversion for mantle viscosity from the retreat of sea-level after deglaciation is successful.



Determination of h_{RSL} by levelling of threshold
Determination of t_i by dating of sample

In northern Canada, available SLIs, mainly **shells and drift** (right), are of minor quality and give only indirect evidence of the former sea level. To infer the retreat of sea level from these samples, a more careful strategy must be chosen.

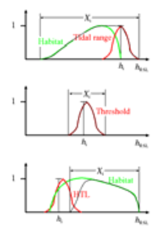


Determination of h_{SLI} by levelling of sample
Determination of t_i by dating of sample
Relating h_{SLI} to h_{RSL} by depositional conditions

Application of fuzzy logic

1. Membership function defining depositional conditions

Depositional conditions are expressed by membership functions, which define the possibility that a specific type of SLI is found at a height with respect to sea level. These membership functions can be specified considering different conditions,such as environment or tidal range. For example (right), the height range of a shallow water shell is enlarged by the mean-tide level (top), the threshold of an isolation basin is shifted by the high-tide level (middle), and the habitat of basal peat is constrained by the high-tide level (bottom).



2. Fuzzyfication of prediction

- Definition of membership function for SLI-height range by applying different fuzzy rules

$$\mathbf{E}_i: \quad X_i \to [0,1]$$
$$h \to \mu_i^{\mathrm{E}}$$

- Prediction of RSL height based on GIA modelling

$$h_i^{\mathrm{P}} = m(x_i, t_i), \ \mathrm{SLI}_i \in S$$

- Prediction of SLI-height range by referencing of membership function

$$\mathbf{P}_i := \mathbf{E}_i + h_i^{\mathrm{P}}: \quad X_i + h_i^{\mathrm{P}} \to [0,1]$$
$$h \to \mu_i^{\mathrm{P}}(h) := \mu_i^{\mathrm{E}}\left(h - h_i^{\mathrm{P}}\right)$$

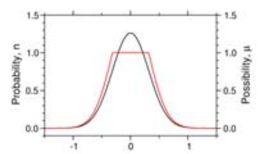
3. Fuzzification of observation (express errors by fuzzy set)

- Normal distribution of errors is described by probability density (black line)

$$n(h, h_i^{\mathrm{O}}, \sigma_i^2) = \frac{1}{\sqrt{2\pi}\,\sigma} \exp\left[-\frac{\left(h - h_i^{\mathrm{obs}}\right)}{2\sigma_i^2}\right]$$

- Consistency principle (here for confidence level of 86% for a=1) after Civanlar & Trussell (1986) gives the corresponding possibility (red line)

$$\mathbf{O}_i: \quad (-\infty, \infty) \to [0,1]$$
$$h \to \mu_i^{\mathrm{O}} := \begin{cases} 1, & \left(h - h_i^{\mathrm{obs}}\right)/\sigma_i < a \\ \exp\left(\frac{a^2}{2} - \frac{\left(h - h_i^{\mathrm{obs}}\right)}{2\sigma_i^2}\right), & \text{otherwise} \end{cases}$$



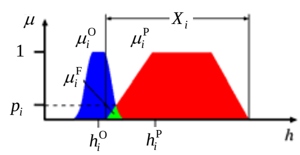
4. Quantification of fit

- Intersection

$$\mathbf{F}_i := \mathbf{P}_i \cap \mathbf{O}_i: \quad X_i^{\mathrm{P}} \to [0,1]$$
$$h \to \mu_i^{\mathrm{F}} := \min\left\{\mu_i^{\mathrm{P}}(h), \mu_i^{\mathrm{O}}(h)\right\}$$

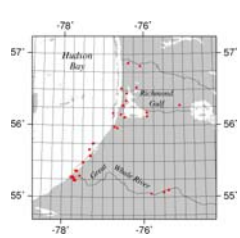
- Defuzzification $\quad p_i := \max_{h \in X_i^{\mathrm{P}}} \mu_i^{\mathrm{F}}(h)$

- Misfit $\quad \chi = \frac{1}{N}\sum_{i=1}^{N}(1 - p_i)$



Example: Decay time of land uplift in the Richmond Gulf, NE Canada

We apply the method for the set of SLIs covering the Richmond Gulf region in NE Canada (right). The area is east of the former glaciation centre of the last Pleistocene glaciation.



The SLIs of this region were collected and analyzed by several authors over the last decades. They consist of 93 samples (sea-level diagram): 29 terrestrial (charcoal, peat), 27 drift (whale bones, drift) shown in red and 27 shallow water shells, 12 ordinary shells shown in blue.

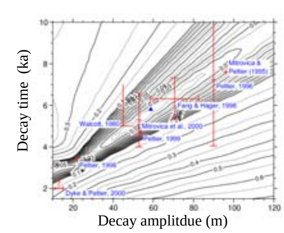
The land uplift after deglaciation roughly follows an exponential decay described by

$$m(\bar{x}, t_i) = A\left[\exp(t_i/T) - 1\right],$$

where A and T are the decay amplitude and decay time, respectively, m is the predicted RSL height, x the average position of the SLIs and t_i the age. We infer A and T by defining suitable membership functions for the different types of SLI, considering tidal range and storm beaches.

Isolines of the **misfit function**, χ, for the inferred parameters. The small blue triangle denotes the **best-fitting parameters (A=58.8 m, T=5.8 ka).**

Error bars and references give the values inferred by other investigators based on different subsets of the SLIs using the conventional procedure.



Sea-level diagram for the Richmond Gulf region. showing the SLIs of the region. SLIs indicating an upper limit are in red and SLIs indicating a lower limit are in blue.

The black line shows the best fitting exponential function and the orange band the range of curves for χ < 0.02.