Non-Gaussianity of an Idealized El Niño Model

Atsuyoshi Manda[1]
[1]Faculty of Fisheries, Nagasaki University, Japan
Correspondence: manda@nagasaki-u.ac.jp

INTRODUCTION

It is widely recognized that forecasts made with atmosphere/ocean models are inherently uncertain. Such uncertainty arises primarily from two sources: imperfect knowledge of the initial conditions and imperfections in the model formulation itself. In addition to studying the intrinsic predictability of the atmosphere-ocean system, efforts have been directed at the quantification or prediction of forecast uncertainty (e.g., Palmer and Hagedorn 2006). The probability density function (PDF) is the most fundamental tool for quantifying the uncertainty.

There is a school of thought that El Niño-Southern Oscillation (ENSO) is nonlinear. Some nonlinear models well reproduce observed features of ENSO (e.g., Zebiak and Cane 1987). Although the issue that ENSO is whether nonlinear or linear process is still in debate, we concentrate the first possibility in this study. Nonlinear processes in the models can significantly change the shape of the PDF. It may exhibit non-zero skewness or bi-modality, which can render the ensemble mean an unlikely state that hardly occurs. The aim of this study is to elucidate how the PDF of an ENSO model is deformed by the nonlinearity through an ensemble experiment.

EXPERIMENT DESCRIPTION

The model used in this study is a version of the delayed oscillator model, developed by Galanti and Tziperman (2000). The model equations are written for the sea surface temperature anomaly (T) and the thermocline depth anomaly (H) in the eastern Pacific as a function of time. All model parameters are the same as those of Galanti and Tziperman (2000).

The model was run for 1000 years to remove all transients (Tziperman et al. 1994), and the final state was used for the initial conditions for the ensemble experiment. Two thousand samples (ensemble members) were used in the ensemble experiment. As initial perturbations, Gaussian noises were added to T and H of each sample. The magnitudes of the Gaussian noises for T and H were set to 1.8×10^{-3} (°C) and 1.3×10^{-2} (m), respectively. These values were 0.1% of the rms values of the time series of T and H obtained from a 150-year run without noises. Each member was run for 150 years to obtain the data for the analysis.

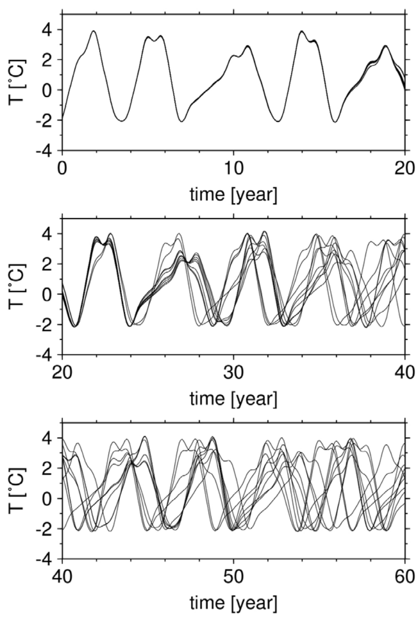

Figure 1 Time evolution of the sea surface temperature anomaly (T) obtained by the ensemble experiment. Only 10 samples are shown for clarity.

TIME EVOLUTION OF PROBABILITY DENSITY FUNCTION

Figure 1 shows the time series of T values for 10 samples obtained from the ensemble experiment. The time series diverges as time advances, due to the chaotic nature of the model (Eccles and Tziperman 2004). Figure 2 shows the time evolution of the marginal PDF of T. Anderson-Darling tests were performed to check the Gaussianity of the PDFs. The null hypothesis that the PDFs are Gaussian is rejected during most of the period of the experiment. The PDFs vary irregularly in time after year 10. Figure 3 shows the time series of the ensemble mean, standard deviation (SD), skewness, and flatness (defined as kurtosis minus 3) of T. The leading Lyapunov exponent estimated from the SDs of T and H is 0.22 year^{-1}. Around year 50, the SD reaches "saturation", when the prediction becomes no better than guesswork (Palmer and Hagedorn 2006). Skewness and flatness change significantly after year 10. To check whether these changes are "apparent variations" due to uncertainty of estimates, 15 additional ensemble experiments were performed. SDs of skewness and flatness were computed using the results of 16 ensemble experiments. From year 5 to 50, the ratios of SDs of skewness and flatness to their means are of order 10^{-3} and 10^{-2}, respectively. This indicates the estimates of skewness and flatness from a single ensemble are robust, and the rapid changes in skewness and flatness shown in Fig. 2 are not apparent variations.

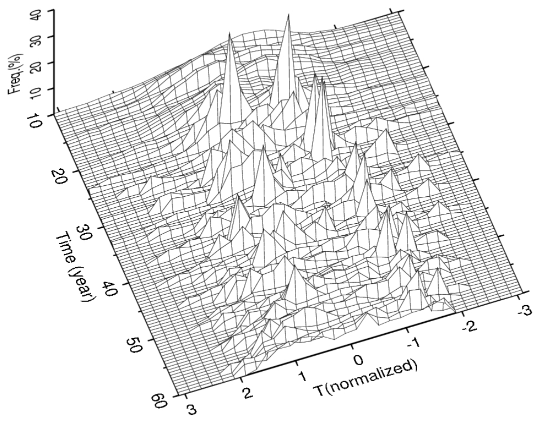

Figure 2 Time evolution of the marginal probability density function of T.

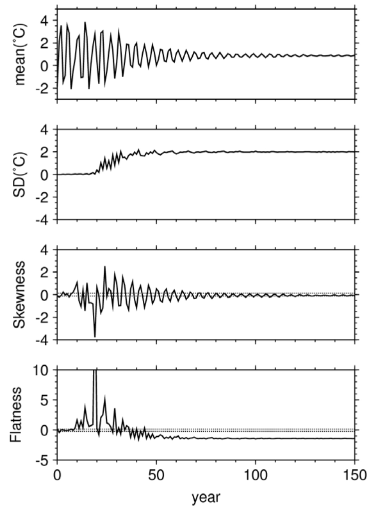

Figure 3 Time evolution of the ensemble mean, standard deviation (SD), skewness and flatness (excess kurtosis) of T.

DISCUSSION

One of the most prominent features revealed in the ensemble experiment is the rapid change in the PDFs (Figs. 2 and 3), the reason for which is discussed in this section. Figure 4 shows the ensemble means of the power spectra of T and their coefficients of variation (CVs), which are defined as standard deviations normalized by the ensemble means. The CVs for most frequencies are smaller than 0.2, which is a 95% significant level of power spectra computed from a single sample. This means that the variations of spectra from their ensemble means are indistinguishable from the statistical noise and indicates that the spectral energy distributions do not vary with the samples in most of the frequency bands.

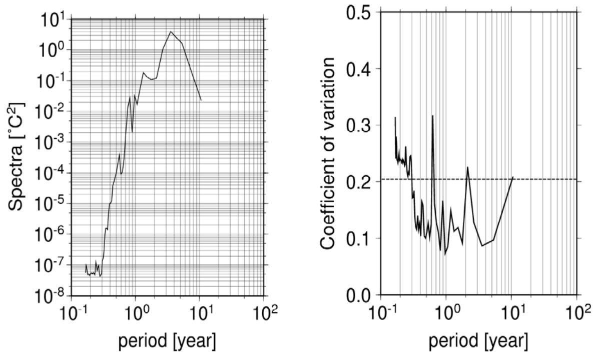

Figure 4 (Left) ensemble mean of the power spectra in variance-preserving form. (Right) coefficient of variation (normalized standard deviation) of the power spectra. Horizontal dashed line indicates the 95% confidence limit of the power spectra of a single sample.

Although the spectral energy distributions hardly vary, the periods of the ENSO events vary with the samples significantly. To investigate the reason for this, a Lorenz map of the period of ENSO events is produced (Fig. 5). Time series of T for 10,000 years obtained from a single model run were used for plotting Fig. 5. Points in Lorenz maps are much scattered, indicating irregular changes in the period. This is attributed to the nonlinear nature of the ENSO. The natural oscillator of the equatorial Pacific coupled ocean-atmosphere system can enter into nonlinear resonance with the seasonal cycle at several periods of the oscillator (Tziperman et al. 1994; Jin et al. 1994). The coexistence of these nonlinear resonances results in chaotic behavior due to the irregular jumps of the system between the different resonances (Tziperman et al. 1995; Eccles and Tziperman 2004). These jumps can lead to the irregular change of ENSO periods.

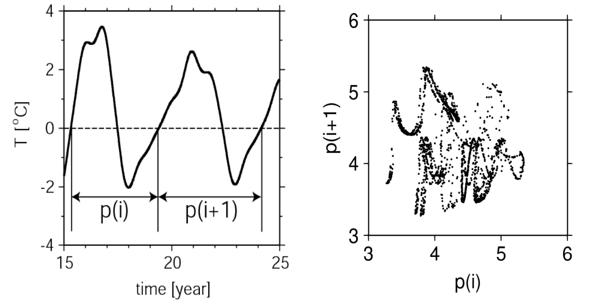

Figure 5 (Left) Definition of the periods of ENSO events. Periods of ith and (i+1)th events are indicated by p(i) and p(i+1), respectively. (Right) Lorenz map of the periods in years.

The irregular changes in periods can deform the shape of the PDF as follows. Suppose we have samples for which their periods are slightly different from each other. The period of each sample at the next event will be different as illustrated in Fig. 5. The period of each sample at the next event varies irregularly with the samples, but this is not a completely random process. The samples tend to "choose" a particular period as shown in Fig. 6. The histograms in Fig. 6 are not Gaussian, indicating that even though the waves of the temperature record can be represented by the same functional form (e.g., sin ωt), the magnitudes of the temperatures sampled at the same time vary with the samples and the PDF deviates from being Gaussian.

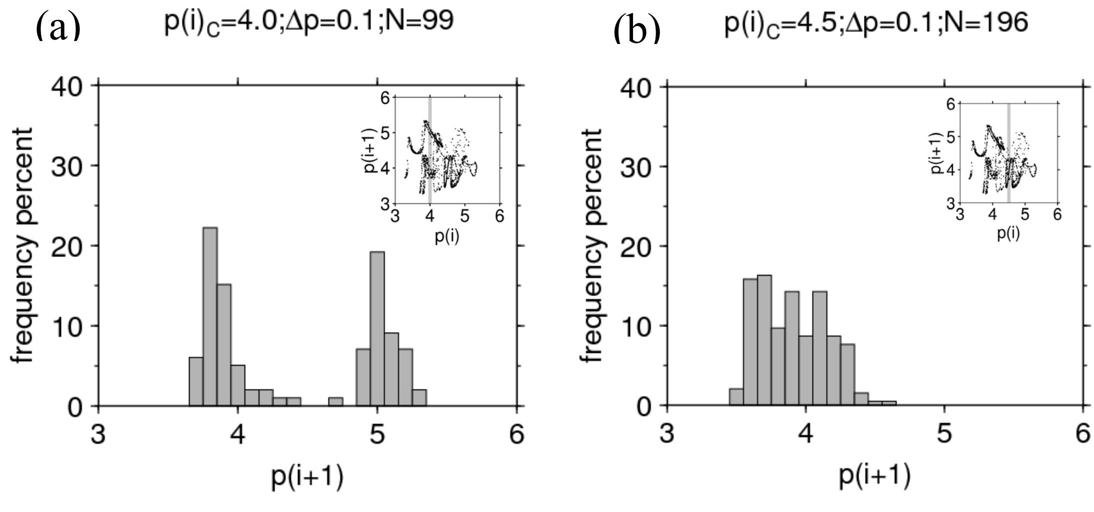

Figure 6 (a) Histograms of p(i+1) in years sampled from the Lorenz map in Fig. 5. Shaded area in the inset panel shows the sampling area. (b) Same as (a) except for the sampling area.

CONCLUSION

The PDF of the delay oscillator model is not Gaussian for most of the period of the ensemble experiment and shows rapid change in time. Irregular changes in the periods of ENSO events due to nonlinear resonance can cause the non-Gaussianity and rapid change of the PDF. A prediction strategy using only mean and variance is typically used for atmosphere/ocean prediction. This strategy is equivalent to approximating the PDF to Gaussian. The quantification of the predictability of the Gaussian approximation of the PDF is one of the crucial predictability issues (Cai et al., 2004). Abramov et al. (2005) shows that relative entropy used in information theory is useful for quantifying the lack of information by using the Gaussian approximation, using a quasi-geostrophic model. They also show that relative entropy can be used for detecting bi-modality, which is important for determining if the ensemble mean is a most probable state. Applying relative entropy to El Niño models would give a better understanding of the forecast uncertainty of the El Niño prediction.

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REFERENCES

Abramov, R., A. Majda and R. Kleeman, 2005: Information theory and predictability for low-frequency variability. J. Atmos. Sci., **62**, 65-87.

Cai, D., K. Haven and A. Majda, 2004: Quantifying predictability in a simple model with complex features, I: the perfect predictability scenario, Stochastics and Dynamics, **4**, 547-569.

Eccles, F. and E. Tziperman, 2004: Nonlinear effects on ENSO's period. J. Atmos. Sci., **61**, 474-482.

Galanti, E. and E. Tziperman, 2000: ENSO’s phase locking to the seasonal cycle in the fast-SST, fast-wave, and mixed-mode regimes. J. Atmos. Sci., **57**, 2936-2950.

Jin, F.-F., D. Neelin and M. Ghil, 1994: El Niño on the devil’s staircase: annual subharmonic steps to chaos. Science, **264**, 70-72.

Palmer, T. and R. Hagedorn, ed., 2006: Predictability of weather and climate. Cambridge University Press, 702pp

Tziperman, E., L. Stone, M. A. Cane and H. Jarosh, 1994: El Niño chaos: overlapping of resonances between the seasonal cycle and the Pacific ocean–atmosphere oscillator. Science, **264**, 72-74.

Tziperman, E., M. A. Cane and S. E. Zebiak, 1995: Irregularity and locking to the seasonal cycle in an ENSO prediction model as explained by the quasi-periodicity route to chaos. J. Atmos. Sci., **52**, 293-306.

Zebiak, S. E. and M. A. Cane, 1987: A model El Niño-Southern Oscillation. Mon. Wea. Rev., **115**, 2262-2278.